Higher twist corrections to the sum rule for semileptonic B decay

Changhao Jin
School of Physics, University of Melbourne
Victoria 3010, Australia

Abstract

The sum rule for charmless inclusive semileptonic B-meson decays allows a theoretically clean and experimentally efficient determination of $|V_{ub}|$. The leading twist contribution to the sum rule is known in QCD. We compute higher twist corrections to the sum rule using the heavy quark effective theory.

A new method to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ has been proposed [1] that takes advantage of the sum rule for charmless inclusive semileptonic B-meson decays $\bar{B} \to X_u \ell \bar{\nu}_\ell$ ($\ell = e$ or μ). The sum rule establishes a clean relationship between $|V_{ub}|$ and the observable

$$S \equiv \int_0^1 d\xi_u \, \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (\bar{B} \to X_u \ell \bar{\nu}_\ell) \tag{1}$$

with the kinematic variable $\xi_u = (q^0 + |\mathbf{q}|)/M_B$ in the B-meson rest frame, where q is the momentum transfer to the lepton pair and M_B denotes the B meson mass. Moreover, this method of determining $|V_{ub}|$ has experimental virtue too. The kinematic variable ξ_u is the most efficient discriminator between $\bar{B} \to X_u \ell \bar{\nu}_\ell$ signal and $\bar{B} \to X_c \ell \bar{\nu}_\ell$ background. A majority of $\bar{B} \to X_u \ell \bar{\nu}_\ell$ events have a value of ξ_u beyond the limit allowed for $\bar{B} \to X_c \ell \bar{\nu}_\ell$ decays with charm in the final state, $\xi_u > 1 - M_D/M_B = 0.65$ with M_D being the D meson mass. Therefore, only a small extrapolation is needed to obtain S.

The charmless inclusive semileptonic decay of the B meson is a light-cone dominated process. The light-cone expansion allows a rigorous and systematic ordering of nonperturbative QCD effects, providing an effective technique for a separation and classification of higher twist (HT) effects [2]. The leading term in this expansion gives the leading twist contribution. HT contributions are contained in the light-cone expansion beyond the leading order. The sum rule at the leading twist order measures the bottomness carried by a B meson. There are no perturbative QCD corrections to the sum rule. Thus the primary hadronic uncertainty and the potential uncertainty of perturbative QCD are eliminated, dramatically reducing the theoretical error on $|V_{ub}|$. This inclusive method is to be contrasted with the determination of $|V_{ub}|$ from the charmless inclusive semileptonic branching fraction of B mesons where the calculation of the total semileptonic decay rate is model dependent or assumes quark-hadron duality, there are uncertainties due to perturbative QCD corrections and, in addition, a larger extrapolation is necessary to extract the total rate if the kinematic cut on a certain observable, such as the charged-lepton energy or the invariant mass of the lepton pair, is applied for the suppression of $b \to c$ background.

Only uncertainties due to HT effects remain in the sum rule. Including the HT contribution ΔHT, the sum rule reads

$$S \equiv \int_0^1 d\xi_u \, \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (\bar{B} \to X_u \ell \bar{\nu}_\ell) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} (1 + \Delta HT) \, . \tag{2}$$

Although HT contributions are expected to be suppressed by powers of $\Lambda_{\rm QCD}^2/M_B^2$ ($\Lambda_{\rm QCD}$ being the QCD scale), a quantitative estimate of them is indispensable for a complete understanding of remaining theoretical uncertainties in this determination of $|V_{ub}|$. In this paper, we investigate HT effects on the sum rule for charmless inclusive semileptonic B decays using the heavy quark effective theory (HQET) [3–5].

Charmless inclusive semileptonic decays of the B meson are induced by the weak interactions. The differential decay rate to lowest order in the weak interactions is

$$d\Gamma = \frac{G_F^2 \, |V_{ub}|^2}{(2\pi)^5 E} L^{\mu\nu} W_{\mu\nu} \frac{d^3 k_\ell}{2E_\ell} \frac{d^3 k_\nu}{2E_\nu} \, . \tag{3}$$

Here E (P), E_ℓ (k_ℓ), and E_ν (k_ν) denote the energies (four-momentums) of the B meson, the charged lepton, and the antineutrino, respectively. The leptonic tensor for the lepton pair is completely determined by the standard electroweak theory since leptons do not have strong interactions:

$$L^{\mu\nu} = 2(k_\ell^\mu k_\nu^\nu + k_\nu^\mu k_\ell^\nu - g^{\mu\nu} k_\ell \cdot k_\nu + i\varepsilon^{\mu\nu}{}_{\alpha\beta} k_\ell^\alpha k_\nu^\beta). \tag{4}$$

The hadronic tensor incorporates all nonperturbative QCD physics for the inclusive semileptonic B decay. It is summed over all hadronic final states and can be expressed in terms of a current commutator taken between the B meson states:

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4y e^{iq\cdot y} \langle B \left| [j_\mu(y), j_\nu^\dagger(0)] \right| B \rangle, \tag{5}$$

where $j_\mu(y) = \bar{u}(y)\gamma_\mu(1-\gamma_5)b(y)$ is the charged weak current for the $b \to u$ transition. We adopt a covariant normalization for one-particle states, i.e., $\langle B(P)|B(P')\rangle = (2\pi)^3 2P^0 \delta^{(3)}(\mathbf{P} - \mathbf{P}')$.

The most general hadronic tensor form that can be constructed is a linear combination of $P_\mu P_\nu$, $P_\mu q_\nu$, $q_\mu P_\nu$, $q_\mu q_\nu$, $\varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$, and $g_{\mu\nu}$, with coefficients being scalar functions $W_a(\eta,\, q^2)$ of the two independent Lorentz invariants, $\nu \equiv q \cdot P/M_B$ and q^2. However, the combination $P_\mu q_\nu - q_\mu P_\nu$ does not contribute since $L^{\mu\nu}(P_\mu q_\nu - q_\mu P_\nu) = 0$. Thus the hadronic tensor must take the form

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{M_B^2} W_2 - i\varepsilon_{\mu\nu\alpha\beta} \frac{P^\alpha q^\beta}{M_B^2} W_3 + \frac{q_\mu q_\nu}{M_B^2} W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^2} W_5\,. \tag{6}$$

Equation (5) shows that $W_{\mu\nu}^* = W_{\nu\mu}$, so W_a, $a = 1, \ldots, 5$ are real. The interesting physics describing the hadron structure and the strong interactions is wrapped up in the five dimensionless real structure functions $W_a(\nu,\, q^2)$, $a = 1, \ldots, 5$ for the unpolarized processes.

In the following we will neglect the masses of the charged lepton and the u-quark. From Eqs. (3) and (6), we obtain the double differential decay rate for $\bar{B} \to X_u \ell \bar{\nu}_\ell$ in the rest frame of the B meson

$$\frac{d^2\Gamma}{d\xi_u dq^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{48\pi^3 M_B} \frac{|\mathbf{q}|^2}{\xi_u} (W_1 3q^2 + W_2 |\mathbf{q}|^2), \tag{7}$$

where

$$|\mathbf{q}| = \frac{1}{2} M_B \xi_u (1 - \frac{q^2}{M_B^2 \xi_u^2}). \tag{8}$$

By integrating Eq. (7) over q^2, one gets the decay distribution of the kinematic variable ξ_u

$$\frac{d\Gamma}{d\xi_u} = \int_0^{M_B^2 \xi_u^2} dq^2 \frac{d^2\Gamma}{d\xi_u dq^2}. \tag{9}$$

Computing the current commutators one obtains from Eq. (5)

$$W_{\mu\nu} = -\frac{1}{\pi}(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta}) \int d^4y e^{iq\cdot y} \left[\partial^\alpha \Delta_u(y)\right] \langle B \left| \bar{b}(0)\gamma^\beta U(0,y) b(y) \right| B \rangle\,, \tag{10}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$. In the above we have used

$$\{u(x), \bar{u}(y)\} = i(\gamma \cdot \partial)i\Delta_u(x-y)U(x,y) \tag{11}$$

with the Wilson link

$$U(x,y) = \mathcal{P}\exp[ig_s \int_y^x dz^\mu A_\mu(z)], \tag{12}$$

$$\Delta_u(y) = -\frac{i}{(2\pi)^3}\int d^4k e^{-ik\cdot y}\varepsilon(k^0)\delta(k^2), \tag{13}$$

where A^μ is the background gluon field and $\varepsilon(x)$ satisfies $\varepsilon(|x|) = 1$ and $\varepsilon(-|x|) = -1$.

The matrix element $\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle$ is the basic building block of the description of inclusive B decays in QCD. In general one can decompose it in the following form:

$$\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle = 2[P^\beta F(y^2, y\cdot P) + y^\beta G(y^2, y\cdot P)], \tag{14}$$

where $F(y^2, y\cdot P)$ and $G(y^2, y\cdot P)$ are functions of the two independent Lorentz scalars, y^2 and $y \cdot P$. The dominant part of the integrand in the hadronic tensor (10) stems from the space-time region near the light cone, with the deviation from the light cone being of the order of the inverse large momentum $y^2 \sim 1/q^2 \sim 1/M_B^2 \to 0$ [2]. The light-cone expansion of the functions $F(y^2, y\cdot P)$ and $G(y^2, y\cdot P)$ in powers of y^2 leads to

$$\begin{aligned}\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle &= 2\left[P^\beta \sum_{n=0}^{\infty}(y^2)^n\mathcal{F}^{(2n+2)}(y\cdot P) + y^\beta\sum_{n=0}^{\infty}(y^2)^n\mathcal{G}^{(2n+4)}(y\cdot P)\right] \\ &= 2\left\{P^\beta\left[\mathcal{F}^{(2)}(y\cdot P) + y^2\mathcal{F}^{(4)}(y\cdot P) + \cdots\right] + y^\beta\left[\mathcal{G}^{(4)}(y\cdot P) + y^2\mathcal{G}^{(6)}(y\cdot P) + \cdots\right]\right\}. \end{aligned}\tag{15}$$

The coefficients $\mathcal{F}^{(2n+2)}(y\cdot P)$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ in the light-cone expansion can be classified by twist. Following the notion of twist introduced by Jaffe and Ji [6], $\mathcal{F}^{(2n+2)}(y\cdot P)$ has twist $2n+2$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ has twist $2n+4$, as from dimension analysis we know that the contribution of the former is suppressed by $(\Lambda_{\rm QCD}/M_B)^{2n}$, and the contribution of the latter is suppressed by $(\Lambda_{\rm QCD}/M_B)^{2n+2}$. We will further discuss the non-local light-cone expansion of matrix elements below.

The twist decomposition for the decay rate thus takes the form

$$d\Gamma = \sum_{n=0}^{\infty} d\Gamma^{(2n+2)}, \tag{16}$$

where

$$d\Gamma^{(2n+2)} = \frac{G_F^2\,|V_{ub}|^2}{(2\pi)^5 E}L^{\mu\nu}W_{\mu\nu}^{(2n+2)}\frac{d^3k_\ell}{2E_\ell}\frac{d^3k_\nu}{2E_\nu} \tag{17}$$

is the twist-$2n+2$ contribution to the decay rate with

$$\begin{aligned} W_{\mu\nu}^{(2n+2)} &= -g_{\mu\nu}W_1^{(2n+2)} + \frac{P_\mu P_\nu}{M_B^2}W_2^{(2n+2)} - i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{M_B^2}W_3^{(2n+2)} \\ &\quad + \frac{q_\mu q_\nu}{M_B^2}W_4^{(2n+2)} + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^2}W_5^{(2n+2)} \end{aligned}\tag{18}$$

$$\begin{aligned} &= -\frac{2}{\pi}(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta})\int d^4y e^{iq\cdot y}\left[\partial^\alpha\Delta_u(y)\right] \\ &\quad \times\left[P^\beta(y^2)^n\mathcal{F}^{(2n+2)}(y\cdot P) + y^\beta(y^2)^{n-1}\mathcal{G}^{(2n+2)}(y\cdot P)\right]. \end{aligned}\tag{19}$$

The leading twist contribution to the sum rule (2) results from $\mathcal{F}^{(2)}(y\cdot P)$ of twist 2 and is known in QCD to be [1]

$$\int_0^1 d\xi_u \, \frac{1}{\xi_u^5}\frac{d\Gamma^{(2)}}{d\xi_u}(\bar{B}\to X_u\ell\bar{\nu_\ell}) = |V_{ub}|^2\frac{G_F^2 M_B^5}{192\pi^3}, \tag{20}$$

which is a consequence of the conservation of the b-quark vector current by the strong interactions. The next-to-leading twist contribution to the sum rule arises from $\mathcal{F}^{(4)}(y\cdot P)$ and $\mathcal{G}^{(4)}(y\cdot P)$ of twist 4. It can be obtained by integrating Eq. (9) over ξ_u with the two relevant structure functions $W_1^{(4)}(\nu,q^2)$ and $W_2^{(4)}(\nu,q^2)$ of twist 4.

We use the operator product expansion and the heavy quark effective theory to compute the twist-4 structure functions. The Wilson link is a gauge dependent operator. It is convenient to use the Fock-Schwinger gauge such that $U(0,y)$ is unity. Since the b quark inside the B meson behaves as almost free due to its large mass, relative to which its binding to the light constituents is weak, one can extract the large space-time dependence

$$b(y) = e^{-im_b v\cdot y}b_v(y), \tag{21}$$

where m_b is the b-quark mass and $v = P/M_B$ is the four-velocity of the B meson. This factorization makes clear why the large scale in matrix elements does not affect the relative size of terms in the light-cone expansion (15). The large scale hidden in matrix elements of b-quark operators is contained in an overall factor $e^{-im_b v\cdot y}$, so reduced matrix elements of the operators containing the rescaled operator b_v involve only momenta of order $\Lambda_{\rm QCD}$, which determine the relative size of terms in the light-cone expansion (15), i.e., schematically

$$\langle B|\bar{b}(0)\gamma^\beta b(y)|B\rangle = e^{-im_b v\cdot y}\langle B|\bar{b}_v(0)\gamma^\beta b_v(y)|B\rangle \sim e^{-im_b v\cdot y}\sum_{n=0}^{\infty}\left(\frac{\Lambda_{\rm QCD}^2}{M_B^2}\right)^n. \tag{22}$$

The rescaled operator for a free b-quark no longer depends on the space-time, so $b(y) = e^{-im_b v\cdot y}b(0)$. In this case all the coefficients $\mathcal{F}^{(2n+2)}(y\cdot P)$ and $\mathcal{G}^{(2n+4)}(y\cdot P)$ in the light-cone expansion (15) vanish except that $\mathcal{F}^{(2)}(y\cdot P) = e^{-im_b v\cdot y}$, because the conservation of the b-quark vector current implies that $\langle B|\bar{b}(0)\gamma^\beta b(0)|B\rangle = 2P^\beta$. The leading-twist sum rule (20) is consistently reproduced in the free quark decay $b\to u\ell\bar{\nu_\ell}$. The conserved vector current $\bar{b}\gamma^\beta b$ is not renormalized by the strong interactions. This explains why there are no perturbative QCD corrections to the sum rule.

A Taylor expansion of the field in a gauge-covariant form relates the bilocal and local operators. This leads to an operator product expansion

$$\bar{b}(0)\gamma^\beta b(y) = e^{-im_b v\cdot y}\bar{b}_v(0)\gamma^\beta b_v(y) = e^{-im_b v\cdot y}\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}y_{\mu_1}\cdots y_{\mu_n}\bar{b}_v(0)\gamma^\beta k^{\{\mu_1}\cdots k^{\mu_n\}}b_v(0), \tag{23}$$

where $k_\mu = iD_\mu = i(\partial_\mu - ig_s A_\mu)$ and the symbol $\{\cdots\}$ means symmetrization with respect to the enclosed indices. Because of the weak dependence of the rescaled operator $b_v(y)$ on y, we attempt to estimate the matrix element of the bilocal operator sandwiched between the B meson states with the truncated y-expansion in Eq. (23). To obtain a twist-4 accuracy it suffices to keep only the first three terms

$$
\begin{aligned}
\langle B|\bar{b}(0)\gamma^\beta b(y)|B\rangle = e^{-im_b v\cdot y}\Bigg[&\langle B|\bar{b}_v(0)\gamma^\beta b_v(0)|B\rangle \\
&+(-i)y_\mu\langle B|\bar{b}_v(0)\gamma^\beta iD^\mu b_v(0)|B\rangle \\
&+\frac{(-i)^2}{2}y_\mu y_\nu\langle B|\bar{b}_v(0)\gamma^\beta iD^{\{\mu}iD^{\nu\}}b_v(0)|B\rangle\Bigg].
\end{aligned}
\tag{24}
$$

In the heavy quark effective theory the QCD b-quark field $b(y)$ is related to its HQET counterpart $h(y)$ by means of an expansion in powers of $1/m_b$:

$$
b(y) = e^{-im_b v\cdot y}\left[1+\frac{i\not{D}}{2m_b}+O\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)\right]h(y). \tag{25}
$$

The effective Lagrangian takes the form

$$
\mathcal{L}_{\rm HQET} = \bar{h}iv\cdot Dh+\bar{h}\frac{(iD)^2}{2m_b}h+\bar{h}\frac{g_sG_{\mu\nu}\sigma^{\mu\nu}}{4m_b}h+O\left(\frac{1}{m_b^2}\right), \tag{26}
$$

where $g_sG^{\mu\nu} = i[D^\mu, D^\nu]$ is the gluon field-strength tensor. At the level of accuracy of the present discussion we take into account only the leading, $1/m_b$ correction to the heavy quark limit $m_b \to \infty$. Relating the matrix elements of the local operators in full QCD in Eq. (24) to those in HQET, it follows that

$$
\begin{aligned}
\langle B|\bar{b}(0)\gamma^\beta b(y)|B\rangle = 2e^{-im_b v\cdot y}\Bigg\{&P^\beta\left[1-y\cdot Pi\frac{5}{3}\frac{m_b}{M_B}E_b-(y\cdot P)^2\frac{1}{3}\frac{m_b^2}{M_B^2}K_b+y^2\frac{1}{3}m_b^2K_b\right] \\
&+y^\beta i\frac{2}{3}m_bM_BE_b\Bigg\},
\end{aligned}
\tag{27}
$$

where $E_b = K_b + G_b$ and K_b and G_b are the dimensionless HQET parameters of order $(\Lambda_{\rm QCD}/m_b)^2$, which are often referred to by the alternate names $\lambda_1 = -2m_b^2K_b$ and $\lambda_2 = -2m_b^2G_b/3$, defined as

$$
\lambda_1 = \frac{1}{2M_B}\langle B|\bar{h}(iD)^2h|B\rangle, \tag{28}
$$

$$
\lambda_2 = \frac{1}{12M_B}\langle B|\bar{h}g_sG_{\mu\nu}\sigma^{\mu\nu}h|B\rangle. \tag{29}
$$

Comparing Eq. (27) with Eq. (15) yields

$$
\mathcal{F}^{(4)}(y\cdot P) = \frac{1}{3}m_b^2K_be^{-im_b v\cdot y}, \tag{30}
$$

$$
\mathcal{G}^{(4)}(y\cdot P) = i\frac{2}{3}m_bM_BE_be^{-im_b v\cdot y}. \tag{31}
$$

We observe that the coefficients $\mathcal{F}^{(4)}(y\cdot P)$ and $\mathcal{G}^{(4)}(y\cdot P)$ of the light-cone expansion (15) are indeed of order $\Lambda_{\rm QCD}^2$ as expected. Substituting Eqs. (30) and (31) in Eq. (19) and integrating by parts, we arrive at

$$
\begin{aligned}
W_{\mu\nu}^{(4)} = \frac{16m_b}{3M_B}\Big\{ & -g_{\mu\nu}M_B^2\Big[\frac{1}{4}m_b(m_b-\nu)K_bX \\
& +E_b\varepsilon(q^0-m_bv^0)\delta(q^2-2m_b\nu+m_b^2) \\
& +m_b(m_b-\nu)K_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2) \\
& +(q^2-2m_b\nu+m_b^2)E_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\Big] \\
& +P_\mu P_\nu m_b^2\left[\frac{1}{2}K_bX+2(K_b+E_b)\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\right] \\
& +i\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta m_bM_BK_b\left[\frac{1}{4}X+\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\right] \\
& +q_\mu q_\nu 2M_B^2E_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2) \\
& +(P_\mu q_\nu+q_\mu P_\nu)m_bM_B\Big[-\frac{1}{4}K_bX-K_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2) \\
& -2E_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\Big]\Big\}, \qquad (32)
\end{aligned}
$$

where $\delta'(x)=\frac{d}{dx}\delta(x)$ and

$$
X=\frac{\partial^2}{\partial q^\mu\partial q_\mu}[\varepsilon(q^0-m_bv^0)\delta(q^2-2m_b\nu+m_b^2)]. \qquad (33)
$$

Comparing Eq. (32) with Eq. (18), we find

$$
\begin{aligned}
W_1^{(4)}(\nu,q^2) = \frac{16}{3}m_bM_B\Big\{ & \frac{1}{4}m_b(m_b-\nu)K_bX+E_b\varepsilon(q^0-m_bv^0)\delta(q^2-2m_b\nu+m_b^2) \\
& +[m_b(m_b-\nu)K_b+(q^2-2m_b\nu+m_b^2)E_b] \\
& \times\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\Big\}, \qquad (34)
\end{aligned}
$$

$$
W_2^{(4)}(\nu,q^2) = \frac{16}{3}m_b^3M_B\left[\frac{1}{2}K_bX+2(K_b+E_b)\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\right], \qquad (35)
$$

$$
W_3^{(4)}(\nu,q^2) = -\frac{16}{3}m_b^2M_B^2K_b\left[\frac{1}{4}X+\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\right], \qquad (36)
$$

$$
W_4^{(4)}(\nu,q^2) = \frac{32}{3}m_bM_B^3E_b\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2), \qquad (37)
$$

$$
W_5^{(4)}(\nu,q^2) = -\frac{16}{3}m_b^2M_B^2\left[\frac{1}{4}K_bX+(K_b+2E_b)\varepsilon(q^0-m_bv^0)\delta'(q^2-2m_b\nu+m_b^2)\right]. \qquad (38)
$$

The twist-4 contribution to the sum rule can be obtained from Eqs. (9), (7), (34) and (35). The result is

$$
\begin{aligned}
\int_0^1 d\xi_u\frac{1}{\xi_u^5}\frac{d\Gamma^{(4)}}{d\xi_u}(\bar{B}\to X_u\ell\bar{\nu}_\ell) = |V_{ub}|^2\frac{G_F^2M_B^5}{192\pi^3}\Big[& \frac{304}{45}K_b+\frac{76}{45}E_b+2\frac{m_b^2}{M_B^2}K_b-\frac{68}{9}\frac{m_b^3}{M_B^3}K_b \\
& -\frac{80}{9}\frac{m_b^3}{M_B^3}E_b-\frac{26}{3}\frac{m_b^4}{M_B^4}K_b+\frac{28}{3}\frac{m_b^4}{M_B^4}E_b+\frac{112}{15}\frac{m_b^5}{M_B^5}K_b \\
& -\frac{32}{15}\frac{m_b^5}{M_B^5}E_b\Big]. \qquad (39)
\end{aligned}
$$

This can serve as an estimate of HT contributions to the sum rule (2).

For the numerical analysis, we need to know the values for the parameters involved. The HQET parameter λ_2 can be extracted from the $B^* - B$ mass splitting: $\lambda_2 = (M_{B^*}^2 - M_B^2)/4 \simeq 0.12$ GeV^2, while λ_1 and m_b are less determined. For the purpose of estimation, we take $\lambda_1 = -0.5$ GeV^2 [7,8] and $m_b = 4.9$ GeV [9]. From Eq. (39), the HT correction to the sum rule (2) is then estimated to be $\Delta HT = 0.012$. This quantitative study shows that HT corrections are at the expected level of $\sim \Lambda_{\rm QCD}^2/M_B^2$.

In summary, we have elaborated a quantitative way of estimating HT contributions in inclusive B decays, which is based on the heavy quark effective theory. As an application we have calculated the twist-4 correction to the sum rule (2) for charmless inclusive semileptonic B decays. Using the sum rule, $|V_{ub}|$ can be determined from a measurement of the weighted integral S. The error on $|V_{ub}|$ due to HT corrections to the sum rule is estimated to be 1%. Combining theoretical cleanliness and experimental efficiency, together with the better understanding of remaining theoretical uncertainties, the sum rule holds high promise of a precise and model-independent determination of $|V_{ub}|$.

ACKNOWLEDGMENTS

Stimulating discussions with Xiao-Gang He and Berthold Stech are gratefully acknowledged. This work was supported by the Australian Research Council.

REFERENCES

[1] C.H. Jin, Mod. Phys. Lett. A 14 (1999) 1163; Phys. Rev. D 62 (2000) 014020.

[2] C.H. Jin, E.A. Paschos, in *Proceedings of the International Symposium on Heavy Flavor and Electroweak Theory*, Beijing, China, 1995, edited by C.H. Chang and C.S. Huang (World Scientific, Singapore, 1996), p. 132; hep-ph/9504375; C.H. Jin, Phys. Rev. D 56 (1997) 2928; C.H. Jin, E.A. Paschos, Eur. Phys. J. C 1 (1998) 523; C.H. Jin, Phys. Rev. D 56 (1997) 7267; Eur. Phys. J. C 11 (1999) 335.

[3] N. Isgur, M.B. Wise, Phys. Lett. B 232 (1989) 113; B 237 (1990) 527.

[4] E. Eichten, B. Hill, Phys. Lett. B 234 (1990) 511; B 243 (1990) 427.

[5] H. Georgi, Phys. Lett. B 240 (1990) 447.

[6] R.L. Jaffe, X. Ji, Nucl. Phys. B 375 (1992) 527.

[7] P. Ball, V.M. Braun, Phys. Rev. D 49 (1994) 2472.

[8] A.S. Kronfeld, J.N. Simone, Phys. Lett. B 490 (2000) 228.

[9] Particle Data Group, D.E. Groom et al., Eur. Phys. J. C 15 (2000) 1.